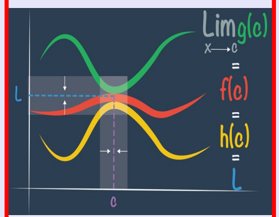
Calculus I

Lecture 24



Feb 19-8:47 AM

Class / Take Home Quiz 21:

Make Sure To do this by wednesday:

$f(x) = (x-5)\sqrt[3]{x^2}$

$f'(x) = \dfrac{5(x-2)}{3\sqrt[3]{x}}$

$f''(x) = \dfrac{10(x+1)}{9x\sqrt[3]{x}}$

1) Domain $(-\infty, \infty)$

2) All intercepts

y-Int $(0,0)$

x-Int $(0,0), (5,0)$

3) Find all critical points $(2, -3\sqrt[3]{4})$

$f'(x)=0$ or undefined $(0,0)$

4) Find all inflection points $(-1,-6)$

Possible inflection pts $(0,0)$

$f''(x)=0$ or und.

May 12-11:08 AM

Class / Take Home Quiz 21:

Make Sure To do this by wednesday:

$f(x)=(x-5)\sqrt[3]{x^2}$

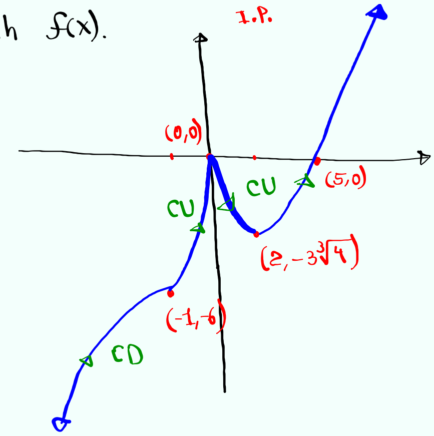
$f'(x)=\dfrac{5(x-2)}{3\sqrt[3]{x}}$

$f''(x)=\dfrac{10(x+1)}{9x\sqrt[3]{x}}$

5) Do a Complete Sign chart

x	$-\infty$		-1		0		2		∞
$f'(x)$		$+$		$+$	o	$-$	•	$+$	
$f''(x)$		$-$	•	$+$	o	$+$		$+$	
$f(x)$			$(-1,-6)$ I.P.		$(0,0)$		$(2,-3\sqrt[3]{4})$		

6) Graph $f(x)$.



May 12-11:08 AM

Mean- Value Theorem

1) $f(x)$ is cont. on $[a,b]$,

2) $f(x)$ is diff. on (a,b)

There is a number c in (a,b) such that

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

Suppose $f(x)=\sqrt{x}$, Interval is $[4,9]$

$f(x)$ is cont. on $[4,9]$

$f'(x)=\dfrac{1}{2\sqrt{x}}$ def. on $(4,9)$ $\Rightarrow f(x)$ is diff. on $(4,9)$

by MVT $\quad f'(c)=\dfrac{f(b)-f(a)}{b-a}$

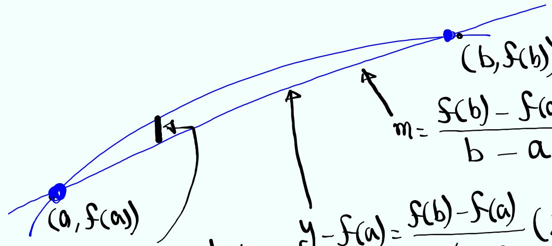
$$\frac{1}{2\sqrt{c}}=\frac{f(9)-f(4)}{9-4} \Rightarrow \frac{1}{2\sqrt{c}}=\frac{3-2}{5}$$

$$\frac{1}{2\sqrt{c}}=\frac{1}{5} \qquad 2\sqrt{c}=5$$

$$\sqrt{c}=\frac{5}{2}$$

6.25 in in $(4,9)$ $\quad c=\left(\frac{5}{2}\right)^2=\frac{25}{4}=\boxed{6.25}$

May 14-9:21 AM



$m = \dfrac{f(b) - f(a)}{b - a}$

vertical distance

$y - f(a) = \dfrac{f(b) - f(a)}{b - a}(x - a)$

$y_{curve} - y_{line}$

$$\boxed{y = \frac{f(b) - f(a)}{b - a}(x - a) + f(a)}$$

$$H(x) = f(x) - \frac{f(b) - f(a)}{b - a}(x - a) - f(a)$$

$H(a) = 0 \quad , \quad H(b) = 0$

$H(x)$ is cont. on $[a, b]$.
$H(x)$ is diff. on (a, b).
$H(a) = H(b)$

} Rolle's Thrm
$H'(c) = 0$

$$H'(x) = f'(x) - \frac{f(b) - f(a)}{b - a}$$

$$f'(c) - \frac{f(b) - f(a)}{b - a} = 0$$

Conclusion of MVT.

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$

May 14-9:28 AM

The product of two positive numbers is 100. → x, y

Find them with minimum sum.

$x > 0$
$y > 0$

$xy = 100$

$y = \dfrac{100}{x}$

$x + y$

$= x + \dfrac{100}{x}$

$f(x) = x + \dfrac{100}{x}$ is minimum.

$f'(x) = 0$ or und.

$f'(x) = 1 - \dfrac{100}{x^2}$

$1 - \dfrac{100}{x^2} = 0$

$x^2 - 100 = 0$

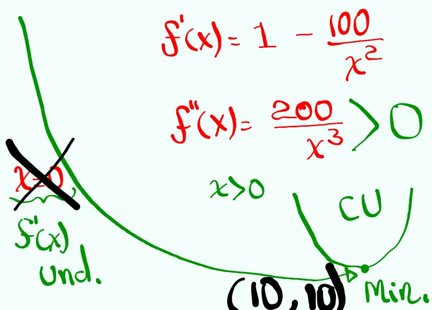
$\boxed{x = 10}$, ~~$x = -10$~~ , ~~$x = 0$~~

$f'(x) = 0$ $\qquad$ $f'(x)$ und.

$f''(x) = \dfrac{200}{x^3} > 0$

$x > 0$

CU

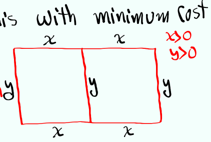


May 14-9:39 AM

A farmer has an area of 1.5 million S.F. in a rectangular field.

He wants to fence it then divide it in half with a fence parallel to of the sides of the rectangle.

How can he do this with minimum cost of fencing?



Area $= 2xy = 1.5$ Million

Fencing $= 4x + 3y$

$x = \frac{1.5\,M}{2y} = \frac{750000}{y}$

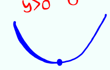
$f(y) = 4\left(\frac{750000}{y}\right) + 3y$

$f(y) = \frac{3000000}{y} + 3y$

$f'(y) = \frac{-300000}{y^2} + 3$

$f''(y) = \frac{6000000}{y^3} > 0$

$y>0$



min.

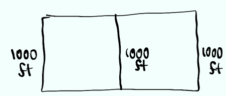
$f'(y) = 0$ or und.

$-\frac{3000000}{y^2} + 3 = 0$

$-3000000 = -3y^2$

$y^2 = 1000000$

$\boxed{y = 1000}$



$4x + 3y = 1.5\,M$

$4x + 3000 = 1500000$

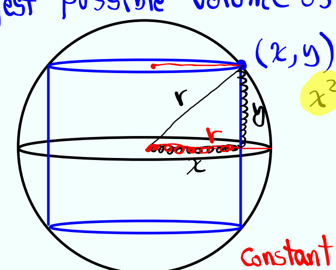
⋮

$\boxed{x =}$

May 14-9:45 AM

A right circular cylinder is inscribed of a sphere of radius r.

Find the largest possible volume of such cylinder.



$x^2 + y^2 = r^2$

$x^2 = r^2 - y^2$

Volume of cylinder

$V = \pi (x)^2 \cdot 2y$

$V = 2\pi x^2 y$

$f(y) = 2\pi (r^2 - y^2) y$

Constant

$f(y) = 2\pi \left[r^2 y - y^3 \right]$

$f'(y)$, $f''(y)$

Max. Volume

Make sure to do this by Monday Morning.

May 14-10:01 AM

$f''(x) = 2 + \cos x$,

$f(0) = -1$, $f(\frac{\pi}{2}) = 0$

Find $f(x)$

$f'(x) = 2x + \sin x + C$

$\boxed{f(x) = x^2 - \cos x + Cx + D}$

$f(0) = \cancel{0^2}^{\to 0} - \cancel{\cos 0}^{\to 1} + \cancel{C(0)}^{\to 0} + D = -1$

$-1 + D = -1$

$\boxed{D = 0}$

$f(\frac{\pi}{2}) = (\frac{\pi}{2})^2 - \cancel{\cos\frac{\pi}{2}}^{\to 0} + C\cdot\frac{\pi}{2} + 0 = 0$

$\frac{\pi^2}{4} + \frac{\pi}{2}C = 0$

LCD = 4

$\pi^2 + 2\pi C = 0$

$2\pi C = -\pi^2$

$C = \frac{-\pi^2}{2\pi} = \boxed{-\frac{\pi}{2}}$

$\boxed{f(x) = x^2 - \cos x - \frac{\pi}{2}x}$

May 14-10:11 AM

$f'''(x) = \cos x$

$f(0) = 1$, $f'(0) = 2$, $f''(0) = 3$

Find $f(x)$.

$f''(x) = \sin x + C$

$f''(0) = \cancel{\sin 0}^{\to 0} + C = 3$

$\boxed{C = 3}$

$f''(x) = \sin x + 3$

$f'(x) = -\cos x + 3x + C$

$f'(0) = -\cancel{\cos 0}^{\to 1} + \cancel{3(0)}^{\to 0} + C = 2$

$\boxed{C = 3}$

$f'(x) = -\cos x + 3x + 3$

$f(x) = -\sin x + \frac{3x^2}{2} + 3x + C$

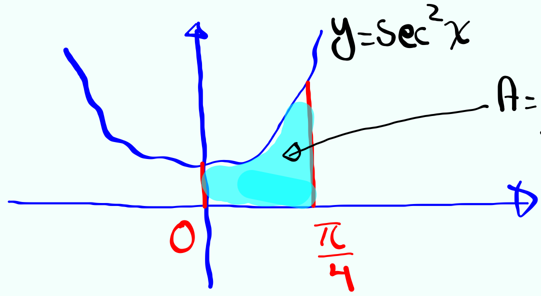
$f(0) = -\sin 0 + \frac{3}{2}(0)^2 + 3(0) + C = 1$

$\boxed{C = 1}$

$\boxed{f(x) = -\sin x + \frac{3}{2}x^2 + 3x + 1}$

May 14-10:22 AM

Find the area below $y = \sec^2 x$, above x-axis from $x = 0$ to $x = \frac{\pi}{4}$.



$$A = \int_0^{\pi/4} [\underbrace{\sec^2 x}_{\text{Top}} - \underbrace{0}_{\text{Bottom}}]\,dx$$

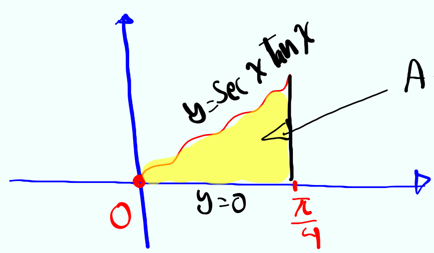
$$= \int_0^{\pi/4} \sec^2 x\,dx = \tan x \Big|_0^{\pi/4}$$

$$= \tan\frac{\pi}{4} - \tan 0$$

$$= \boxed{1}$$

May 14-10:32 AM

Find the area below $f(x) = \sec x \tan x$, above x-axis from $x = 0$ to $x = \frac{\pi}{4}$.



$$A = \int_0^{\pi/4} (\sec x \tan x - 0)\,dx$$

$$= \int_0^{\pi/4} \sec x \tan x\,dx$$

$$= \sec x \Big|_0^{\pi/4}$$

$$= \sec\frac{\pi}{4} - \sec 0$$

$$= \boxed{\sqrt{2} - 1}$$

May 14-10:39 AM

Find f_{ave} for $f(x) = \sec^2 x$ on $[0, \frac{\pi}{4}]$

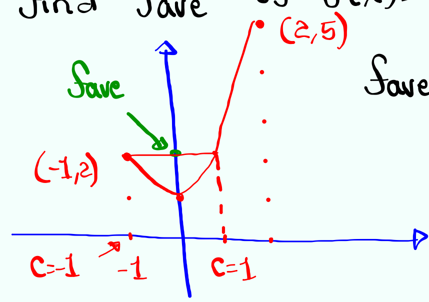
$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{\frac{\pi}{4} - 0}\int_0^{\pi/4} \sec^2 x\,dx$$

$$= \frac{4}{\pi} \cdot \tan x \Big|_0^{\pi/4} = \text{- - - - -} = \boxed{\frac{4}{\pi}}$$

May 14-10:45 AM

Find f_{ave} of $f(x) = 1 + x^2$ on $[-1, 2]$.



$$f_{ave} = \frac{1}{2-(-1)}\int_{-1}^{2} (1+x^2)\,dx$$

$$= \frac{1}{3}\left(x + \frac{x^3}{3}\right)\Big|_{-1}^{2}$$

$$= \frac{1}{3}\left[\left(2 + \frac{8}{3}\right) - \left(-1 - \frac{1}{3}\right)\right]$$

$$= \frac{1}{3}\left[2 + \frac{8}{3} + 1 + \frac{1}{3}\right]$$

$$= \frac{1}{3}\left[3 + \frac{9}{3}\right] = \frac{1}{3}[3 + 3]$$

$$= \frac{1}{3} \cdot 6 = \boxed{2} \checkmark$$

Mean-Value theorem for integration

If $f(x)$ is cont. on $[a,b]$, then there is c in $[a,b]$ such that $f(c) = f_{ave}$.

$1 + c^2 = 2 \qquad c^2 = 1 \qquad \boxed{c = \pm 1}$

May 14-10:49 AM

$f(x) = \sqrt{x}$, $[0,4]$

Find all c in $[0,4]$ such that $f(c) = f_{ave}$

$$f_{ave} = \frac{1}{4-0}\int_0^4 \sqrt{x}\,dx = \frac{1}{4}\int_0^4 x^{1/2}\,dx$$

$$= \frac{1}{4}\left[\frac{x^{3/2}}{3/2}\right]\Bigg|_0^4$$

$$= \frac{1}{4}\cdot\frac{2}{3}\; x\sqrt{x}\,\Big|_0^4$$

$$= \frac{1}{6}\left[4\sqrt{4} - 0\sqrt{0}\right] = \frac{1}{6}\cdot 8 = \frac{8}{6} = \frac{4}{3}$$

$f(c) = f_{ave}$

$\sqrt{c} = \frac{4}{3}$ $\boxed{c = \frac{16}{9}}$ is in $[0,4]$

May 14-11:02 AM

Find all numbers c in $[2,5]$ such that $f(c) = f_{ave}$ for $f(x) = x^2 - 6x + 9$.

Polynomial $\to$ Cont. $(-\infty,\infty)$

$$f_{ave} = \frac{1}{5-2}\int_2^5 (x^2 - 6x + 9)\,dx$$

$$= \frac{1}{3}\left[\frac{x^3}{3} - 3x^2 + 9x\right]\Bigg|_2^5$$

$$= \frac{1}{3}\left[\left(\frac{125}{3} - 75 + 45\right) - \left(\frac{8}{3} - 12 + 18\right)\right]$$

$$= \frac{1}{3}\left[\frac{125}{3} - 30 - \frac{8}{3} - 6\right]$$

$$= \frac{1}{3}\left[\frac{117}{3} - 36\right] = \frac{1}{3}\left[39 - 36\right] = 1$$

Now $f(c) = f_{ave}$

$c^2 - 6c + 9 = 1 \to c^2 - 6c + 8 = 0$

$(c-4)(c-2) = 0$

$\boxed{c=4}$ $\boxed{c=2}$

are in $[2,5]$

May 14-11:09 AM